Apparatus for Studying the Relationship Between Pressure and Volume of a Gas

As $P\ (h)$ increases, $V$ decreases.
Boyle’s Law

\[ P \propto \frac{1}{V} \]

\[ P \times V = \text{constant} \]

\[ P_1 \times V_1 = P_2 \times V_2 \]

Constant temperature
Constant amount of gas
A sample of chlorine gas occupies a volume of 946 mL at a pressure of 726 mmHg. What is the pressure of the gas (in mmHg) if the volume is reduced at constant temperature to 154 mL?

\[ P \times V = \text{constant} \]

\[ P_1 \times V_1 = P_2 \times V_2 \]

\[ P_1 = 726 \text{ mmHg} \quad P_2 = ? \]

\[ V_1 = 946 \text{ mL} \quad V_2 = 154 \text{ mL} \]

\[ P_2 = \frac{P_1 \times V_1}{V_2} = \frac{726 \text{ mmHg} \times 946 \text{ mL}}{154 \text{ mL}} = 4460 \text{ mmHg} \]
Variation in Gas Volume with Temperature at Constant Pressure

As $T$ increases, $V$ increases.
Variation of Gas Volume with Temperature at Constant Pressure

\[ V \propto T \]

\[ V = \text{constant} \times T \]

\[ \frac{V_1}{T_1} = \frac{V_2}{T_2} \]

Charles’ & Gay-Lussac’s Law

Temperature **must** be in Kelvin

\[ T (K) = t (^0C) + 273.15 \]
A sample of carbon monoxide gas occupies 3.20 L at 125 °C. At what temperature will the gas occupy a volume of 1.54 L if the pressure remains constant?

\[ \frac{V_1}{T_1} = \frac{V_2}{T_2} \]

\( V_1 = 3.20 \text{ L} \quad V_2 = 1.54 \text{ L} \)

\( T_1 = 398.15 \text{ K} \quad T_2 = ? \)

\( T_1 = 125 \text{ (°C)} + 273.15 \text{ (K)} = 398.15 \text{ K} \)

\[ T_2 = \frac{V_2 \times T_1}{V_1} = \frac{1.54 \text{ L} \times 398.15 \text{ K}}{3.20 \text{ L}} = 192 \text{ K} \]
Avogadro’s Law

$V \alpha$ number of moles $(n)$

$V = \text{constant} \times n$

$V_1 / n_1 = V_2 / n_2$

Constant temperature
Constant pressure

$3\text{H}_2(g)$  +  $\text{N}_2(g)$  →  $2\text{NH}_3(g)$

3 molecules  +  1 molecule  →  2 molecules

3 moles  +  1 mole  →  2 moles

3 volumes  +  1 volume  →  2 volumes
Ammonia burns in oxygen to form nitric oxide (NO) and water vapor. How many volumes of NO are obtained from one volume of ammonia at the same temperature and pressure?

\[4\text{NH}_3 + 5\text{O}_2 \rightarrow 4\text{NO} + 6\text{H}_2\text{O}\]

1 mole \(\text{NH}_3\) \(\rightarrow\) 1 mole \(\text{NO}\)

At constant \(T\) and \(P\)

1 volume \(\text{NH}_3\) \(\rightarrow\) 1 volume \(\text{NO}\)
Summary of Gas Laws

Boyle’s Law

Increasing or decreasing the volume of a gas at a constant temperature

$$P = \left(\frac{nRT}{V}\right) \quad nRT \text{ is constant}$$
Charles Law

Heating or cooling a gas at constant pressure

$$ V = \left( \frac{nR}{P} \right) T \quad \frac{nR}{P} \text{ is constant} $$

Heating or cooling a gas at constant volume

$$ P = \left( \frac{nR}{V} \right) T \quad \frac{nR}{V} \text{ is constant} $$
Avogadro’s Law

Dependence of volume on amount of gas at constant temperature and pressure

Avogadro’s Law

\[ V = \left( \frac{RT}{P} \right) n \]

\( \frac{RT}{P} \) is constant
Ideal Gas Equation

Boyle’s law: \( P \alpha \frac{1}{V} \) (at constant \( n \) and \( T \))

Charles’ law: \( V \alpha T \) (at constant \( n \) and \( P \))

Avogadro’s law: \( V \alpha n \) (at constant \( P \) and \( T \))

\[ V \alpha \frac{nT}{P} \]

\[ V = \text{constant} \times \frac{nT}{P} = R \frac{nT}{P} \quad R \text{ is the gas constant} \]

\[ PV = nRT \]
The conditions 0 \degree C and 1 atm are called **standard temperature and pressure (STP)**.

Experiments show that at STP, 1 mole of an ideal gas occupies 22.414 L.

\[
PV = nRT
\]

\[
R = \frac{PV}{nT} = \frac{(1 \text{ atm})(22.414 \text{L})}{(1 \text{ mol})(273.15 \text{ K})} = 0.082057 \text{ L} \cdot \text{atm} / (\text{mol} \cdot \text{K})
\]
What is the volume (in liters) occupied by 49.8 g of HCl at STP?

\[ PV = nRT \]
\[ V = \frac{nRT}{P} \]

Temperature:
- \( T = 0^\circ C = 273.15 \, K \)

Pressure:
- \( P = 1 \, atm \)

Moles of HCl:
- \( n = 49.8 \, g \times \frac{1 \, mol \, HCl}{36.45 \, g \, HCl} = 1.37 \, mol \)

Volume:
- \( V = \frac{1.37 \, mol \times 0.0821 \, \frac{L \cdot atm}{mol \cdot K} \times 273.15 \, K}{1 \, atm} = 30.7 \, L \)
Argon is an inert gas used in lightbulbs to retard the vaporization of the filament. A certain lightbulb containing argon at 1.20 atm and 18 °C is heated to 85 °C at constant volume. What is the final pressure of argon in the lightbulb (in atm)?

\[ PV = nRT \quad n, V \text{ and } R \text{ are constant} \]

\[ \frac{nR}{V} = \frac{P}{T} = \text{constant} \]

\[ \frac{P_1}{T_1} = \frac{P_2}{T_2} \]

\[ P_2 = P_1 \times \frac{T_2}{T_1} = 1.20 \text{ atm} \times \frac{358 \text{ K}}{291 \text{ K}} = 1.48 \text{ atm} \]
PROBLEM: Boyle’s apprentice finds that the air trapped in a J tube occupies 24.8 cm³ at 1.12 atm. By adding mercury to the tube, he increases the pressure on the trapped air to 2.64 atm. Assuming constant temperature, what is the new volume of air (in L)?

PLAN:  

SOLUTION:  

\[
V_1 = 24.8 \text{ cm}^3 \quad V_2 = \text{unknown}
\]

\[
\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}
\]

\[
P = \frac{V_1}{V_2}
\]

\[
V_2 = \frac{P_1 V_1}{P_2} = 0.0248 \text{ L}
\]

\[
\frac{1.12 \text{ atm}}{2.64 \text{ atm}} = 0.0105 \text{ L}
\]
**Sample Problem 5.3  Applying the Pressure-Temperature Relationship**

**PROBLEM:** A steel tank used for fuel delivery is fitted with a safety valve that opens when the internal pressure exceeds 1.00x10^3 torr. It is filled with methane at 23°C and 0.991 atm and placed in boiling water at exactly 100°C. Will the safety valve open?

**PLAN:**
- P₁(atm) and T₁ and T₂(°C)
- 1 atm = 760 torr
- K = °C + 273.15
- P₁(torr) and T₁ and T₂(K)
- x T₂/T₁
- P₂(torr)

**SOLUTION:**
- P₁ = 0.991 atm
- P₂ = unknown
- T₁ = 23°C
- T₂ = 100°C

\[
\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}
\]

\[
\frac{0.991 \text{ atm} \times 760 \text{ torr}}{1 \text{ atm}} = 753 \text{ torr}
\]

\[
P_2 = P_1 \frac{T_2}{T_1} = 753 \text{ torr} \times \frac{373 \text{ K}}{296 \text{ K}} = 949 \text{ torr}
\]
Sample Problem 5.4  Applying the Volume-Amount Relationship

**PROBLEM:** A scale model of a blimp rises when it is filled with helium to a volume of 55 dm$^3$. When 1.10 mol of He is added to the blimp, the volume is 26.2 dm$^3$. How many more grams of He must be added to make it rise? Assume constant T and P.

**PLAN:** We are given initial $n_1$ and $V_1$ as well as the final $V_2$. We have to find $n_2$ and convert it from moles to grams.

<table>
<thead>
<tr>
<th>$n_1$ (mol) of He</th>
<th>$x \frac{V_2}{V_1}$</th>
<th>$n_2$ (mol) of He</th>
<th>subtract $n_1$</th>
<th>mol to be added</th>
<th>$x M$</th>
<th>g to be added</th>
</tr>
</thead>
</table>

**SOLUTION:**

We have $n_1 = 1.10$ mol, $V_1 = 26.2$ dm$^3$, $V_2 = 55.0$ dm$^3$. The volume is given by $V = nRT$, where $R$ is the gas constant. The problem states that $T$ and $P$ are constant, so we can set up the following equation:

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$$

Solving for $n_2$, we get:

$$n_2 = n_1 \frac{V_2}{V_1}$$

Substituting the given values:

$$n_2 = 1.10 \frac{55.0}{26.2} = 2.31 \text{ mol}$$

To convert moles of He to grams, we use the molar mass of He, which is approximately 4.003 g/mol He:

$$4.003 \text{ g He/mol He} \times 2.31 \text{ mol} = 9.24 \text{ g He}$$

Therefore, 9.24 g of He must be added to make the blimp rise.
Sample Problem 5.5  Solving for an Unknown Gas Variable at Fixed Conditions

**PROBLEM:** A steel tank has a volume of 438 L and is filled with 0.885 kg of O₂. Calculate the pressure of O₂ at 21°C.

**PLAN:** V, T and mass, which can be converted to moles (n), are given. We use the ideal gas law to find P.

**SOLUTION:**

\[ n = \frac{0.885 \text{ kg}}{32.00 \text{ g mol}^{-1}} = 27.7 \text{ mol O}_2 \]

\[ P = \frac{nRT}{V} = \frac{24.7 \text{ mol} \times 0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} 	imes 294.15 \text{K}}{438 \text{ L}} = 1.53 \text{ atm} \]
Sample Problem 5.6 Using Gas Laws to Determine a Balanced Equation

PROBLEM: The piston-cylinders below depict a gaseous reaction carried out at constant pressure. Before the reaction, the temperature is 150K; when it is complete, the temperature is 300K.

New figures go here.

Which of the following balanced equations describes the reaction?

1. \( A_2 + B_2 \rightarrow 2AB \)
2. \( 2AB + B_2 \rightarrow 2AB_2 \)
3. \( A + B_2 \rightarrow AB_2 \)
4. \( 2AB_2 \rightarrow A_2 + 2B_2 \)

PLAN: We know \( P, T, \) and \( V \), initial and final, from the pictures. Note that the volume doesn’t change even though the temperature is doubled. With a doubling of \( T \) then, the number of moles of gas must have been halved in order to maintain the volume.

SOLUTION: Looking at the relationships, the equation that shows a decrease in the number of moles of gas from 2 to 1 is equation (3).